Relative Velocity

Relative velocity describes motion with respect to a specific reference frame.

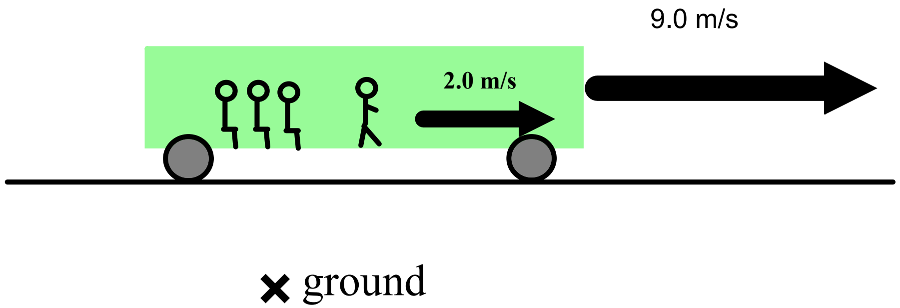
Example

If a dog swims in a fast-flowing river, his velocity could be given relative to the moving water, to the ground, to the tree, to the man, etc.

When velocities are along the same line, simple addition or subtraction is sufficient to obtain relative velocity.

Sample Problem

A passenger walks to the front of a moving train. People on the train see the passenger walking with a velocity of +2.0 m/s. Suppose the train is moving with a velocity of +9.0 m/s relative to an observer standing on the ground.

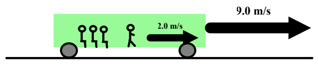


What would be the velocity of the passenger relative to the ground?

There is a special labelling system for relative velocity problems.

Label each velocity using two subscripts:

- the ***first*** refers to the object in question
- the ***second*** refers to the object relative to which the velocity is measured



× ground

V_{pt} - the velocity of the ***p***assenger relative to the ***t***rain = +2.0 m/s

V_{tg} - the velocity of the ***t***rain relative to the ***g***round = +9.0 m/s

V_{pg} - the velocity of the ***p***assenger relative to the ***g***round

the two middle letters must be the same

$$V_{pg} = V_{pt} + V_{tg}$$

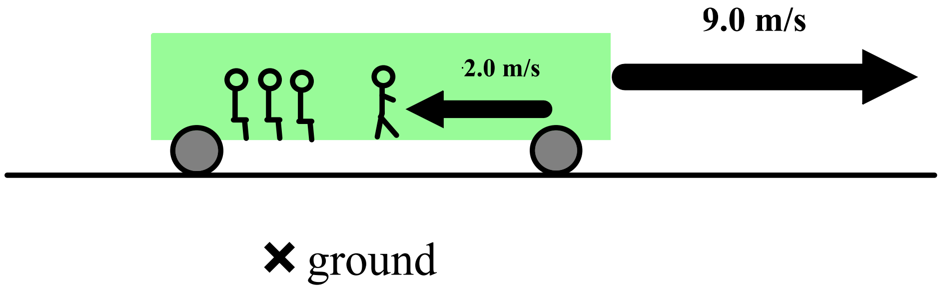
first — last

$$V_{pg} = 2.0 \text{ m/s} + 9.0 \text{ m/s}$$

$$V_{pg} = +11.0 \text{ m/s}$$

Sample Problem

If the passenger had been walking toward the rear of the train, what would be the passenger's velocity relative to the ground?



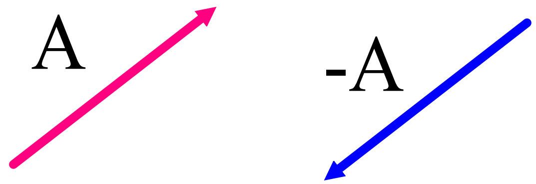
$$V_{pg} = V_{pt} + V_{tg}$$

$$V_{pg} = -2.0\ \text{m/s} + 9.0\ \text{m/s}$$

$$V_{pg} = +7.0\ \text{m/s}$$

Negative Vectors

Multiplying a vector by a negative changes the direction of the vector.



For any two objects, A and B, the velocity of A relative to B has the same magnitude but opposite direction as the velocity of B relative to A.

$$V_{BA} = - V_{AB}$$

Relative Velocity

Sample Problem 1

1. Two trains are passing each other on adjacent tracks. Train A is moving east with a speed of 13 m/s relative to the ground, and train B is traveling west with a speed of 28 m/s relative to the ground.

   a) What is the velocity of train A relative to train B?

   b) What is the velocity of train B relative to train A?

Answer

1. Two trains are passing each other on adjacent tracks. Train A is moving east with a speed of 13 m/s, and train B is traveling west with a speed of 28 m/s.

   a) What is the velocity of train A relative to train B?

$V_{AG} = +13\ m/s$

$V_{BG} = -28\ m/s$

$V_{AB} = V_{AG} + V_{GB}$

$V_{AB} = V_{AG} - V_{BG}$

$V_{AB} = 13 - (-28)$

$V_{AB} = 41\ m/s$

$\therefore V_{AB} = 41\ m/s$

The velocity of train A relative to train B is 41 m/s east.

   b) What is the velocity of train B relative to train A?

$V_{BA} = V_{BG} + V_{GA}$

$V_{BA} = V_{BG} - V_{AG}$

$V_{BA} = -28 - (13)$

$V_{BA} = -41\ m/s$

$\therefore V_{BA} = 41\ m/s$

OR $V_{BA} = -V_{AB} = -(41\ \frac{m}{s})$

$= -41 m/s$

The velocity of train B relative to train A is 41 m/s west.

Sample Problem 2

A cruise ship is traveling relative to the water at a speed of 5.0 m/s due south. Relative to the ship, a passenger walks toward the back of the ship at a speed of 1.5 m/s.

a) What is the magnitude and direction of the passenger's velocity relative to the water?

b) How long does it take for the passenger to walk a distance of 27 m on the ship?

Answer

a) What is the magnitude and direction of the passenger's velocity relative to the water?

P- passenger
S- ship
W- water

$V_{sw} = -5.0$ m/s
$V_{ps} = 1.5$ m/s

$$V_{pw} = V_{ps} + V_{sw}$$

$$V_{pw} = 1.5 + (-5.0)$$

$$V_{pw} = -3.5 \text{ m/s}$$

The velocity of the passenger with respect to the water is 3.5 m/s south.

b) How long does it take for the passenger to walk a distance of 27 m on the ship?

$$V = \frac{d}{t}$$

$$t = \frac{d}{V} \leftarrow V_{ps}$$

$$t = \frac{27}{1.5}$$

$$t = 18 \text{ s}$$

It will take the passenger 18 s to walk a distance of 27 m on the ship.

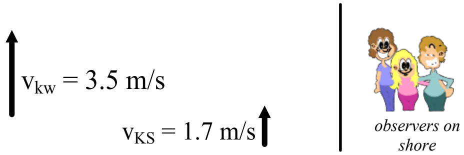
Textbook

pg. 110, Questions 21

21. A kayaker paddles upstream in a river at 3.5 m/s relative to the water. Observers on shore note that he is moving at only 1.7 m/s upstream. Determine the velocity of the current in the river.

21. A kayaker paddles upstream in a river at 3.5 m/s relative to the water. Observers on shore note that he is moving at only 1.7 m/s upstream. Determine the velocity of the current in the river.

current (water relative to shore) V_{ws}?



$$V_{ws} = V_{wk} + V_{ks}$$

$$V_{ws} = -V_{kw} + V_{ks}$$

$$V_{ws} = -(3.5) + (1.7)$$

$$V_{ws} = -\ 1.8 \text{ m/s}$$

The velocity of the current is 1.8 m/s downstream.

$$V_{ks} = V_{kw} + V_{ws}$$

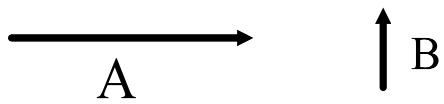
$$V_{ks} - V_{kw} = V_{ws}$$

$$1.7 \text{ m/s} - 3.5 \text{ m/s} = V_{ws}$$

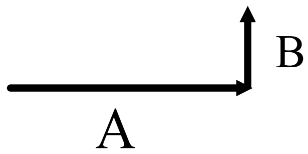
$$-1.8 \text{ m/s} = V_{ws}$$

Adding Vectors Graphically

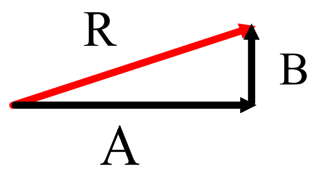
The Tip-to-Tail Method



To add two vectors graphically, place the tail of the second vector at the tip of the first vector.



Draw the resultant vector from the tail of the first vector to the tip of the second vector.

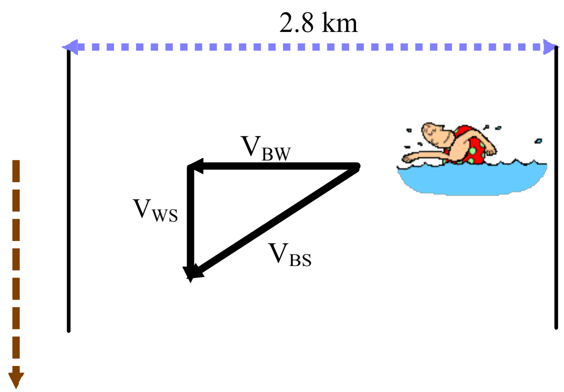


Sample Problem 3

Bubba Newton is capable of swimming at a speed of 1.4 m/s in still water. He starts to swim directly across a 2.8 km wide river. However, the current is 0.91 m/s, and it carries Bubba downstream.

a) How long does it take Bubba to cross the river?

b) How far downstream will Bubba be upon reaching the other side of the river?



Answer

a) $t = \frac{d}{v} = \frac{2.8 \times 10^3}{1.4}$

$= 2.0 \times 10^3\ s$

b) $d = vt$

$d = 0.91\,(2.0 \times 10^3)$

$d = 1.8 \times 10^3\ m$

$1.8 \times 10^3 m$

Each second, Bubba travels 1.4 m across the river and 0.91 m downstream. These two motions happen at the same time.

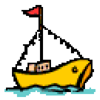
In no way does the downstream velocity change Bubba's velocity across the river.

River Crossing Problems

If the relative velocities of an object are not along the same line, ***vector addition*** must be used.

Sample Problem

Imagine a boat crossing a river.



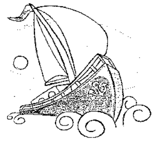
River Boat Simulator

There are three relative velocities that must be taken into consideration.

V_{bw} : the velocity of the ***b***oat with respect to the ***w***ater (heading/still water)

V_{ws} : the velocity of the ***w***ater with respect to the ***s***hore (current)

V_{bs} : the velocity of the ***b***oat relative to the ***s***hore



Investigation – Go with the Flow

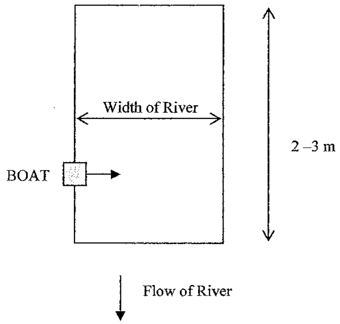
In this investigation, you will simulate the motion of a boat travelling across a river. This process will help your understanding of relative velocities.

Problem

Test your prediction about the point where a boat will come ashore after crossing a river.

Equipment

1 battery-powered truck
1 battery-powered green flatbed
1 stopwatch
1 protractor
1 meter stick
2 – 3 m of craft paper
masking tape



Procedure

1. A battery-powered truck will serve as the boat. Design and carry out a procedure to determine the boat's speed. Record your data and calculations.
2. The craft paper will serve as your river. Measure and record the width of the river.
3. Use the procedure from Step 2 to determine the speed of the green flatbed.

Step 1!

4. Tape the green flatbed to the craft paper. This vehicle will ensure the river flows at a constant velocity.

5. Make the following predictions about the motion of the boat when the river is flowing. Assume that the boat is pointed directly across the river.

   a) Predict whether the motion of the river will affect the time it takes for the boat to travel from one bank to another.

   b) Predict where the boat will come ashore on the opposite riverbank.

6. Test your predictions according to the following procedures.

   a) Measure the time it takes for the boat to cross the river when the river is not flowing.

   b) Mark the boat's starting point with a piece of masking tape on the floor. Using the data you have collected thus far, calculate where the boat should come ashore on the opposite riverbank. Mark the boat's landing point with a piece of masking tape on the floor. Draw a line connecting the boat's starting point and landing point on the craft paper.

   c) Start the river flowing. Start a stopwatch when you start the boat, and time the crossing. Observe the crossing to see how well the boat followed the predicted path.

Analyze and Conclude

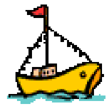
1. Did the boat move in the direction in which it was pointed? Explain.

2. Did your observations support your prediction about the effect of the motion of the river and the time it took the boat the cross the river?

Generally, there are two possible situations.

Situation One

- an object heads straight across a river and actually travels in a direction at some angle relative to the shore



River Boat Simulator

$$V_{bs} = V_{bw} + V_{ws}$$

first last

The velocities that need to be added are perpendicular to each other. Therefore, algebraic addition won't work. We will need to find the vector sum/resultant of the two velocities.

$\therefore$ We will use the Pythagorean Theorem and tan θ.

NOTE

Perpendicular velocities are independent of each other!!!

The time it takes a boat, swimmer, canoe, dog, duck or fish to cross a river depends only on velocity of the object relative to the water and the width of the river.

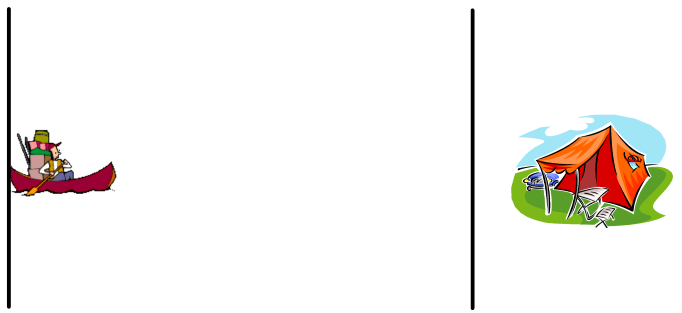
Sample Problem 4 (*Text*: page 105)

A canoeist is planning to paddle to a campsite directly across a river that is 624 m wide. The velocity of the river is 2.0 m/s south. In still water, the canoeist can paddle at a speed of 3.0 m/s. If the canoeist points his canoe straight across the river, toward the east:

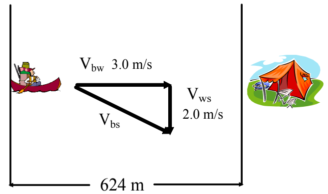
a) How long will it take him to reach the opposite river bank?

b) Where will he land relative to the campsite?

c) What is the velocity of the canoe relative to his initial position on the river bank?



Answer



a) The time it takes to cross the river depends only on the velocity of the canoe relative to the water and is independent of the motion of the water.

$$t = \frac{d}{v}$$

$$t = \frac{624}{3.0}$$

$$t = 2.1 \times 10^2 \text{ s}$$

b) During the 2.1×10^2 s that the canoeist was paddling, the river current was carrying her south, down the river. To find the distance down the river that she landed, find the distance she would travel at the velocity of the current.

$$d = vt$$

$$d = -2.0(2.1 \times 10^2)$$

$$d = -4.2 \times 10^2 \text{ m}$$

He will land 4.2×10^2 m south of the campsite.

c) $V_{CS} = V_{CW} + V_{WS}$

$$(V_{cs})^2 = (V_{cw})^2 + (V_{ws})^2$$

$$(V_{cs})^2 = (3.0)^2 + (-2.0)^2$$

$$(V_{cs})^2 = 13$$

$$V_{cs} = 3.6 \text{ m/s}$$

$$\tan\theta = \frac{2.0}{3.0}$$

$$\tan\theta = 0.6667$$

$$\theta = 34^\circ$$

$$\alpha = 90^\circ - 34^\circ$$

$$\alpha = 56^\circ$$

The velocity of the canoeist relative to his initial position on the shore is 3.6 m/s [S56ºE].

Situation Two

- an object heads in a direction at some angle to the shore and actually travels straight across the river

October 19, 2009

Bell Work

Problems to Solve

You have ~ 10 minutes to answer these questions.

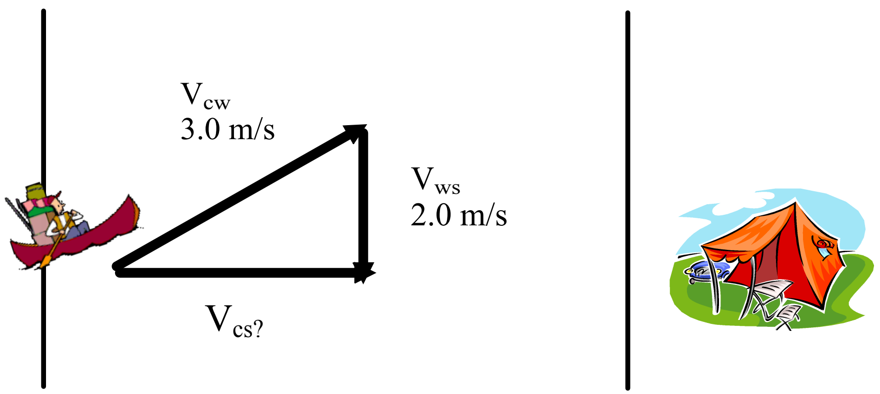
Pick up the handout from the green table at the front of the room. Answer the questions on the handout.

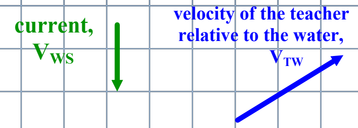
Once the answers have been checked fold the handout and put it in your Hilroy scribbler.

Sample Problem 5

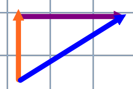
A canoeist wants to head his canoe in such a direction that he will actually travel straight across the river to a campsite. (Use same values as last question.)

a) In what direction must he point his canoe?
b) Find the magnitude of his velocity relative to the shore.
c) How long will it take the canoeist to paddle to the campsite?

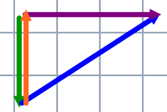




V_{TW} can be resolved into perpendicular components.

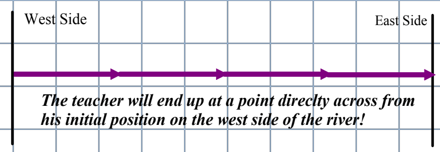


Add V_{WS} and V_{TW} using the tip-to-tail method.

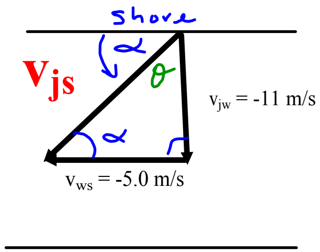


The velocity of the current and the vertical component of the teacher's velocity relative to the water are equal in magnitude and opposite in direction.

The sum of the two velocities, V_{WS} and V_{TW}, is equal to the horizontal component of the teacher's velocity relative to the water!



22. A jet-ski speeds across a river at 11 m/s relative to the water. The jet ski's heading is due south. The river is flowing west at a rate of 5.0 m/s. Determine the jet-ski's velocity relative to the shore.



$V_{JS}^2 = V_{JW}^2 + V_{WS}^2$

$V_{JS}^2 = (-11)^2 + (-5.0)^2$

$V_{JS}^2 = 146$

$V_{JS} = 12 \text{ m/s}$

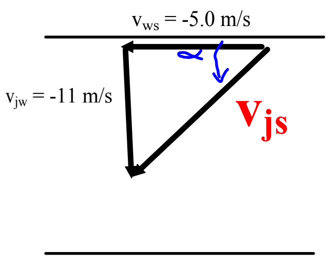
$\tan\theta = \frac{5.0}{11}$

$\theta = 24°$

$\alpha = 90° - 24° = 66°$

The velocity of the jet-ski relative to shore is:

12 m/s [W66°S]



$\tan\theta = \frac{11}{5.0}$

$\tan\theta = 2.2$

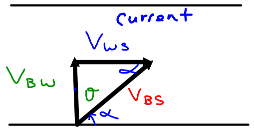
$\theta = 66°$

48. Jerry watches a stick float downstream in a river and notes that it moves 12 m[E] in 2.0×10^1 s. His friend Ben is starting on the south side of the river and is going to swim across. In still water Ben knows that he can swim with a speed of 1.7 m/s. What is Ben's velocity relative to the shore? If the river is 1.5 km wide, how long will it take Ben to cross the river? How far downstream will he land?

This info can be used to find the velocity of the current, V_{ws}.

$$V_{ws} = \frac{d}{t} = \frac{12}{2.0 \times 10} = 0.60 \frac{m}{s}$$

$\therefore V_{ws} = 0.60 \frac{m}{s}$ east



$$V_{BS}^2 = V_{BW}^2 + V_{ws}^2$$
$$V_{BS}^2 = 1.7^2 + 0.60^2$$
$$V_{BS} = 1.8 m/s$$

$$\tan\theta = \frac{0.60}{1.7} = 0.35229$$
$$\theta = 19°$$
$$\alpha = 90° - 19°$$
$$\alpha = 71°$$

$\therefore$ Ben's velocity relative to the shore is:

1.8 m/s [E71°N]

b) Ben's velocity across the river is 1.7 m/s.

Width of river $= 1.5 \times 10^3$ m

$$t = \frac{d}{v} = \frac{1.5 \times 10^3}{1.7} = 8.8 \times 10^2 s$$

c) $d = V_{ws} t$

$$d = 0.60(8.8 \times 10^2)$$
$$d = 5.3 \times 10^2 m$$

River Crossing Problem - Situation 2

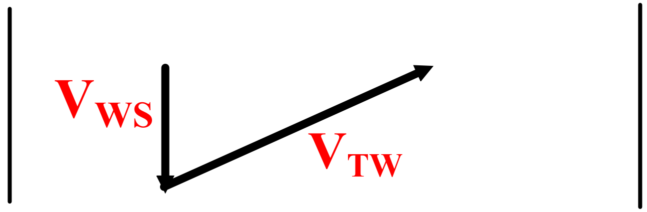
(*page 117*)

24. A physics teacher is on the west side of a small lake and wants to swim across and end up at a point directly across from his starting point. He notices that there is a current in the lake and that a leaf floating by him travels 4.2 m[S] in 5.0 s. He is able to swim 1.9 m/s in calm water.

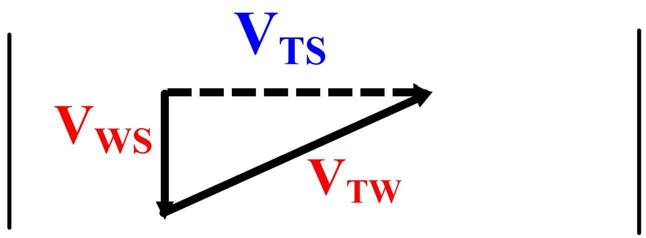
(a) What direction will he have to swim in order to arrive at a point directly across from his position?

The velocity vectors that need to be added are the velocity of the current, V_{ws}, and the velocity of the teacher relative to the water, V_{TW}.

These vectors can be added using the tip-to-tail method.



The resultant is equal to the velocity of the teacher relative to the shore, V_{TS}.



NOTE: For this type of problem, the components of the resultant are not perpendicular.

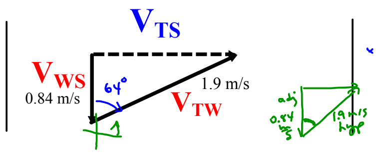
(page 117)

24. A physics teacher is on the west side of a small lake and wants to swim across and end up at a point directly across from his starting point. He notices that there is a current in the lake and that a leaf floating by him travels 4.2 m[S] in 5.0 s. He is able to swim 1.9 m/s in calm water.

(a) What direction will he have to swim in order to arrive at a point directly across from his position?

(b) Calculate his velocity relative to the shore.

(c) If the lake is 4.8 km wide, how long will it take him to cross?



$* \; V_{ws} = \frac{d}{t} = -\frac{4.2}{5.0} = -0.84 \frac{m}{s}$

∴ The velocity of the current is 0.84 m/s South.

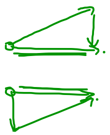
a) $\cos\theta = \frac{0.84}{1.9}$

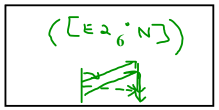
$\cos\theta = 0.4421$

$\theta = 64°$

The teacher should head [N64°E].

Note: If you added the velocity of the current to the velocity of the teacher relative to the water, you would get this diagram.





b) $V_{TS} = 1.9 \sin 64°$

$V_{TS} = 1.7 \text{ m/s}$

∴ The teacher's velocity relative to the shore is 1.7 m/s, east.

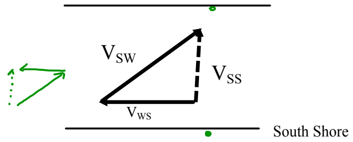
c) $t = \frac{d}{V_{TS}}$ ← You need to use the velocity directed EAST (across the river).

$t = \frac{4.8 \times 10^3}{1.7} = 2.8 \times 10^3 \text{ s}$

25. A swimmer is standing on the south shore of a river that is 120 m wide. He wants to swim straight across and knows that he can swim 1.9 m/s in still water. He drops a stick in the water and finds that it floats with the current to a point 24 m west in 30.0 s. *(page 110)*

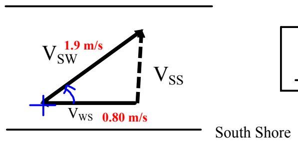
(a) Determine the direction in which the swimmer should head so that he lands directly across the river on the north bank.

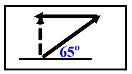
(b) If he follows your advice, determine how long it will take him to reach the far shore.



a) $V_{ws} = \frac{d}{t} = \frac{-24}{30.0} = -0.80 \frac{m}{s}$

∴ The velocity of the current is $0.80 \frac{m}{s}$ west.





$\cos\theta = \frac{0.80}{1.9}$

$\cos\theta = 0.42105$

$\theta = 65°$

∴ The swimmer must head [E 65° N]

b) To calculate the time it takes the swimmer to cross the river, you need to find V_{ss} - the velocity that is directed across the river!

$V_{ss} = 1.9 \sin 65°$

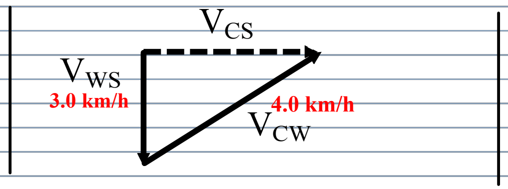
$V_{ss} = 1.7\ m/s$

$t = \frac{d}{v} = \frac{120}{1.7} = 71\ s$

29. A canoeist wants to travel straight across a river that is 0.10 km wide. However, there is a strong current moving downstream with a velocity of 3.0 km/hr. The canoeist can maintain a velocity relative to the water of 4.0 km/hr.

(a) In what direction should the canoeist head to arrive at a position on the other shore directly opposite to his starting position?

(b) How long will the trip take him?



a) $\cos\theta = \frac{3.0}{4.0}$

$\cos\theta = 0.75$

$\theta = 41°$

The canoeist must head upstream at an angle of 41° with respect to the riverbank.

b) $V_{cs}^2 = V_{cw}^2 - V_{ws}^2$

$V_{cs}^2 = (4.0)^2 - (3.0)^2$

$V_{cs}^2 = 16 - 9.0$

$V_{cs}^2 = 7.0$

$V_{cs} = 2.646$ km/h

$t = \frac{d}{V_{cs}} = \frac{0.10}{2.646} = 0.038$ h

It will take the canoeist 0.038 h (1.4×10^2 s) to cross the river.